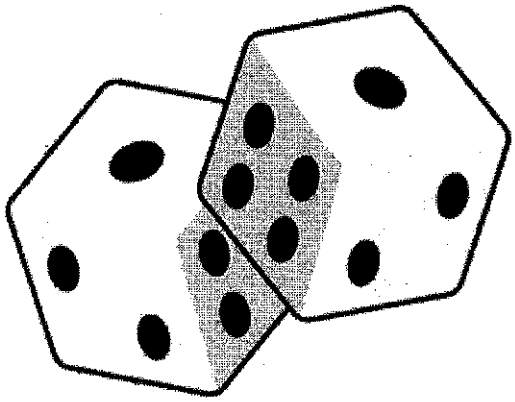
Advanced Mathematical Decision Making

Unit 2
Probability



The Carnival Project!

Group Members: _____ _____ _____

Game : _____

Proposal: Group turns in a proposal that includes but is not limited to the following. The proposal should be written in complete sentences. • What is the game and how is it played? • How do you win? What can you win? • How much are you planning on charging? • Include a diagram of what it will look like • What supplies will you need to make the game? • Why do you think your game will make the biggest profit?	15 points total Due on 9-1	
Theoretical Probability: • List all possible outcomes for your game (including not winning at all) and determine the theoretical probability of each outcome • Include math/justifications for how you determined each probability	10 points total Due on 9-15	
Expected Value: • Using cost of each prize, determine the expected value of playing your game **once.** Remember that not winning at all is an outcome and needs to be included in your expected value. • Include math/justifications for how you determined this	10 points total Due on 9-15	
Carnival Day! • You are set up ON TIME • Someone is running your game at all times • Your game is well constructed • Your game is organized • You kept accurate records of each play of your game • It is clear how to play, how to win and what the prizes are • You have a minimum of 30 trials	20 points total Due on 9-19	
Experimental Probability: • Determine the experimental probability of each event from carnival day • Include math/justifications for how you determined this	10 points total Due on 9-21	

COMPARE: Theoretical v. Experimental probability • Compare what you thought would occur (theoretical) with what actually occurred (experimental). Did the probabilities increase? Decrease? Or stay the same? Give reasons for why this may have occurred.	10 points total Due on 9-21	
COMPARE: Expected Value v. Actual Profit • Take your expected value and multiply it by the number of trials that **actually occurred** on carnival day • To determine your actual profit take the money your earned on carnival day and subtract the money spent on prizes (show your math) • Compare that expected number to your actual profit. Was your profit higher or lower than your expected value? Give reasons why this may have occurred.	10 points total Due on 9-21	
Report/Reflection (2 parts) Part 1: Report (10 points) You are trying to sell your game to an executive at Six-Flags for them to use in all of their theme parks. Type a letter to the president of sales describing why he/she should purchase your game. Be convincing by using data and expected value/profit to back up your point. Part 2: Reflection (5 points) • What went well for you in this project? What didn't go so well? • If you were to do this project again, what would you change? • What would you recommend to next semester's seniors before starting this project • What did you learn from doing this project?	15 points total Due on 9-21	

ENSURING A HEALTHY POPULATION

In this unit, you will be investigating the mathematical and ethical dilemmas of *Ensuring a Healthy Population.* Consider the following quote[1] from a report on health insurance:

> *"Millions of people in the United States go without health insurance each year. The gaps in our health insurance system affect people of all ages, races and ethnicities, and income levels; however, those with the lowest incomes face the greatest risk of being uninsured. Being uninsured affects people's access to needed medical care and their financial security. The access barriers facing uninsured people mean they are less likely to receive preventive care, are more likely to be hospitalized for conditions that could have been prevented, and are more likely to die in the hospital than those with insurance. The financial impact also can be severe. Uninsured families struggle financially to meet basic needs, and medical bills can quickly lead to medical debt."*

What questions does this bring up for you about health care and health insurance?

[1] Source: http://files.kff.org/attachment/the-uninsured-a-primer-key-facts-about-health-insurance-and-the-uninsured-in-america-primer

The Unit Question

Buying health insurance is a way to protect yourself from having to pay for costly medical procedures that become necessary from routine or unexpected health incidents.

You pay a certain amount every month, called the *premium*, to the insurance company. Most of the time, if you don't need medical care, the insurance company just takes your money and they pay you nothing.

Sometimes, however, you have a medical need. When that happens, the insurance company has to pay your expenses (surgery, visit to the doctor, or whatever the incident is), and they generally have to pay you much more in that month than you paid as a premium. So, you pay a smaller amount each month so that when you have a large medical bill (which you may not be able to afford), the insurance company will pay for you. The costs[1] for medical procedures and treatments to cure the conditions that threaten our "citizens'" health are below. As you can see, they are quite costly!

Medical Procedure or Treatment	Cost
Heart Surgery	*$150,000*
Diabetes Treatment	*$50,000*
Cancer Treatment	*$100,000*
Organ Transplant	*$300,000*

In the long run, insurance companies take in more money in premiums than they pay out in claims, or they wouldn't be in business.

What is a fair price to charge for health insurance?

Your Assignment

1. As a group, decide how much you will charge each citizen for health insurance each year. You might start by determining prices for the "citizens" in your own group and then extending to considering the whole "country."
2. Who benefits most from the pricing scheme your group developed?
3. What factors did your group consider when deciding on a price to charge individuals for health insurance? Were some factors more important than others? Why?
4. What additional information, if any, would help your group make a better decision about pricing?

[1] Costs are approximated based on actual data

Rolling for Citizens[1]

In this unit, you will be looking into the mathematical (and other) issues of health care and health insurance. In order to better understand how health care and health insurance are experienced by a variety of different types of people, you will become a random "citizen" in your classroom "country."

To find out the characteristics of your citizen, simply follow the directions for each category – gender, age, ethnicity, yearly income, and health profile. Be sure to record your results on your ***Rolling for Citizens Worksheet***. Many of the problems in this unit will deal directly with your citizen and the other citizens in your country.

GENDER *roll 1 die*	*If the result is* **2, 4, 6**	**Female**
	If the result is **1, 3, 5**	**Male**

AGE *roll 2 dice and add numbers*	*If the result is* **7, 9**	**Choose age under 20**
	If the result is **2, 5, 6**	**Choose age 20 – 40**
	If the result is **3, 4, 8**	**Choose age 40 – 60**
	If the result is **10, 11, 12**	**Choose age over 60**

ETHNICITY *roll 2 dice and multiply numbers*	*If the result is* **6, 8, 9, 10, 12, 15, 16, 18, 20, 24**	**White**
	If the result is **1, 4, 25**	**Black**
	If the result is **2, 3, 5**	**Hispanic**
	If the result is **30**	**Asian**
	If the result is **36**	*Choose from:* **Native American** **Pacific Islander** **Other Ethnicity**

[1] *Sources:*
(1) U.S. Bureau of the Census. Income, Poverty, and Health Insurance Coverage in the United States: 2006–2011.
(2) https://en.wikipedia.org/wiki/Demography_of_the_United_States

YEARLY INCOME *roll 2 dice and add numbers*	*If you are White, and...*	
	...the result is **2, 3**	**$10,000**
	...the result is **4, 5**	**$20,00**
	...the result is **6**	**$35,000**
	...the result is **7**	**$50,000**
	...the result is **8, 9**	**$75,000**
	...the result is **10, 11, 12**	**$100,000 +**
	If you are Black, and...	
	...the result is **2, 3, 4, 11**	**$10,000**
	...the result is **5, 6, 12**	**$20,00**
	...the result is **7**	**$35,000**
	...the result is **8**	**$50,000**
	...the result is **9**	**$75,000**
	...the result is **10**	**$100,000 +**
	If you are Latino, Native American, Pacific Islander, or Other and...	
	...the result is **2, 3, 10**	**$10,000**
	...the result is **4, 5, 11**	**$20,00**
	...the result is **6, 12**	**$35,000**
	...the result is **7**	**$50,000**
	...the result is **8**	**$75,000**
	...the result is **9**	**$100,000 +**
	If you are Asian, and...	
	...the result is **2, 3, 4**	**$10,000**
	...the result is **5**	**$20,00**
	...the result is **6**	**$35,000**
	...the result is **7**	**$50,000**
	...the result is **8**	**$75,000**
	...the result is **9, 10, 11, 12**	**$100,000 +**

HEALTH PROFILE roll 2 dice and add numbers	If your yearly income is $49,000 or lower, and...	
	...the result is **1, 2**	You are in good health! You are at a ***lower than normal*** risk for ***cancer***.
	...the result is **3, 4, 5**	The health care services in your area are below standard. You are at a ***higher than normal*** risk for ***organ transplant***.
	...the result is **6**	There is a history of heart problems in your family. You are at a ***higher than normal*** risk for ***heart surgery***.
	...the result is **7, 8**	You can't always afford to eat healthy food options. You are at a ***higher than normal*** risk for ***diabetes***.
	...the result is **9**	You are fortunate to have found a good doctor who helps you prevent issues! You are at a ***normal*** risk for ***diabetes***.
	...the result is **10, 11, 12**	You work multiple jobs and lead a stressful life. You are at a ***higher than normal*** risk for ***heart surgery***.
	If your yearly income is $50,000 or higher, and...	
	...the result is **1, 2, 3**	You have the time and services to maintain a healthy lifestyle. You are at a ***lower than normal*** risk for ***heart surgery***.
	...the result is **4**	You have been born with organ disease. You are at a ***higher than normal*** risk for ***organ transplant***.
	...the result is **5, 6, 7**	The health services in your area are top notch! You are at a ***lower than normal*** risk for ***diabetes***.
	...the result is **8, 9**	There is no history of cancer in your family! You are at a ***lower than normal*** risk for ***cancer***.
	...the result is **10, 11**	You are busy with work and family, but exercise when you can. You are at a ***normal*** risk for ***diabetes***.
	...the result is **12**	You have slightly higher than normal blood pressure. You are at a ***normal*** risk for ***heart surgery***.

Rolling for Citizens Worksheet

Record information about your citizen below. Be sure to save this sheet for your records. You will need it throughout the unit.

MY CITIZEN	
	The gender of my citizen is...
	The age of my citizen is...
	The ethnicity of my citizen is...
	The yearly income of my citizen is...
	The health profile of my citizen is...

Insurance at Home

Talk with someone at home to see what ideas and questions they have about health insurance. You might start by telling them a little bit about what you have done in class so far. Also, ask them:

1. What do you know about how health insurance works?
2. What have been your experiences with health insurance?
3. What questions do you have about health insurance?

Finally, share our unit question with them and ask:

4. What ideas they have about how to set a fair price for health insurance.

Complete a short write up of your conversation that identifies who you talked with and a summarizes their responses to each of the four questions above.

Pricing Possibilities

Continue working in your group to develop a fair price for health insurance.

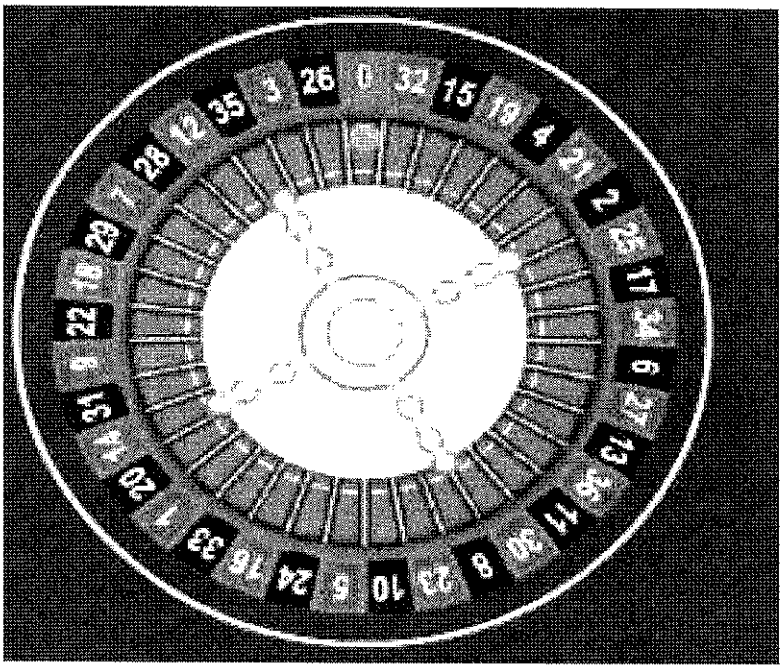
1. Share the ideas for setting a fair price that you used in *"Insurance at Home"*.
2. Discuss these ideas and decide, as a group, on what you think is the fairest way to price health insurance.
3. Write this "best method" in such a way that another group would be able to price health insurance using this method.

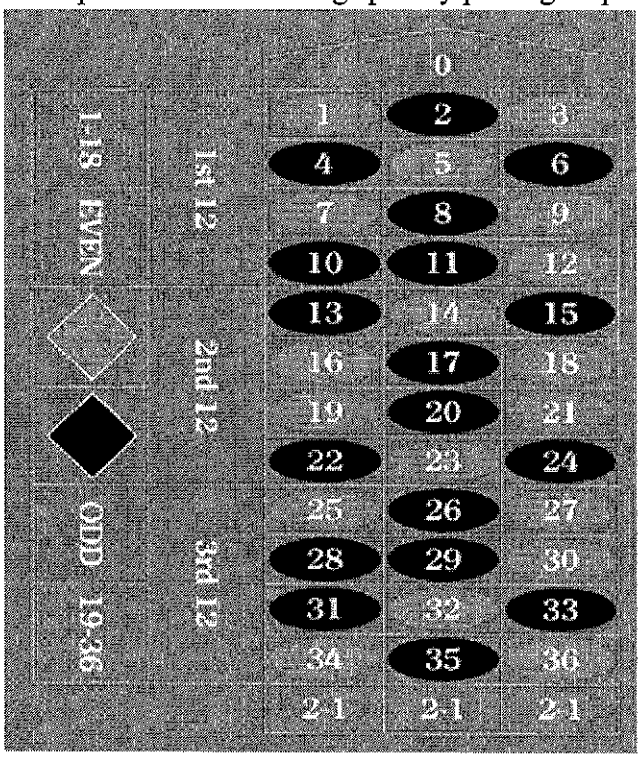
Unit 2: Probability
Probability and Money: Betting at Roulette

In the past you have learned about calculating probability of certain events. We are going to learn why probability matters by linking it to something we all care about- Money.

ROULETTE

Roulette is a game of chance played at casinos. The game is played by a ball being spun on a wheel of 37 numbers with alternating colors of red and black. Players make bets prior to the ball being spun by placing chips on the table shown to the right.





The Bets:

Bet	**Payout**
Single number	35 to 1
Two adjoining numbers (placed on line in between each)	17 to 1
"Street" Any 3 horizontal numbers (placed at end of line)	11 to 1
"Corner" (Placed in the intersection of the 4 numbers)	8 to 1
"Column" (Placed at bottom of the column)	2 to 1
"Dozen" Each set of 12 (placed to the right of the set)	2 to 1
1-18 or 19-36	1 to 1
Red or Black	1 to 1
Even or Odd	1 to 1

The payout refers to how much money you will get back if your bet is correct. For example if you bet $5 on Red and it is Red you will win $5. But if you bet $5 on the number 17 and it lands on the number 17 you will win $595. If you bet on something and do not win then you will lose the money that you initially bet.

The payouts increase as the probability of the event happening decreases. So it is harder to correctly bet on a single number BUT you stand to make more money.

THEORETICAL PROBABILITY: This is what we THINK should happen. Remember there are 37 spaces on the roulette wheel.

A) A single number

B) Two adjoining numbers

C) Street

D) Corner

E) Column

F) Dozen

G) 1-18 or 19-36

H) Red or Black

I) Even or Odd

EXPECTED VALUE: Expected value of an event tells you how much you expect to win for each probability. So the payout of the event times the probability of that event actually occurring. This represents the actual money amount you would expect for each event. ($ times Probability)

A) A single number

B) Two adjoining numbers

C) Street

D) Corner

E) Column

F) Dozen

G) 1-18 or 19-36

H) Red or Black

I) Even or Odd

EXPECTED VALUE WHEN YOU HAVE TO PAY TO PLAY: Remember that this isn't 'free money'. We have to pay in order to play. So even though I may expect to win $0.94 I had to PAY $1 to play. So we have to subtract the payment from the expected winnings.

A) A single number

B) Two adjoining numbers

C) Street

D) Corner

E) Column

F) Dozen

G) 1-18 or 19-36

H) Red or Black

I) Even or Odd

<u>QUESTION:</u> Why are they all negative?

LET'S PLAY!!!!!

To make the math a little easier you are going to pick one bet (single number, two adjoining, street, corner, column, dozen, 1-18, 19-36, red/black, oven/odd) and you are going to bet that EVERY TIME. No changing of bets.

I am going to give you 40 chips (for the 40 times we are going to play). Before each spin, place your chip on your bet.

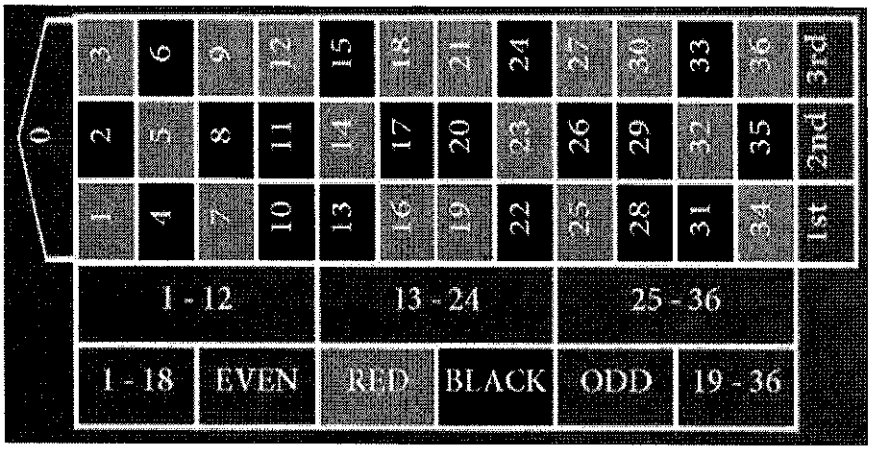
If you win: you keep your chip AND you get to take your winnings from the house.
Ex. If you bet on 34/35 and the 35 comes up. You keep your one chip bet AND collect 17 chips.

If you lose: You lose your chip to the house.

a) What is your bet?

b) What is your payout if you win?

c) What is the theoretical probability of your bet?



d) Multiply your probability times 40. This represents the **number of times** you **expect to win** (round to the nearest whole number).

e) How many times do you need to win to make a profit? (This would be at least one more win than what you expected).

Use the table below to keep track of each spin that occurs

REFLECTION:

1) How much money do you have?

2) Subtract this from $40 (your starting amount). How much did you win (or lose)?

3) How many **times** did you win?

4) The **EXPERIMENTAL PROBABILITY** is the probability that actually occurred. Take the amount of times you won divided by the total times we played (40). Find the experimental probability of your bet. (percentage)

5) Was the experimental probability higher/lower/same as the theoretical probability?

6) **FREE SPACE JOURNALING:** Write about your experience with this simulation. It can be about what you learned, your opinions on gambling, thoughts about what you would have done different. Etc.

CHANCE AND PREDICTION

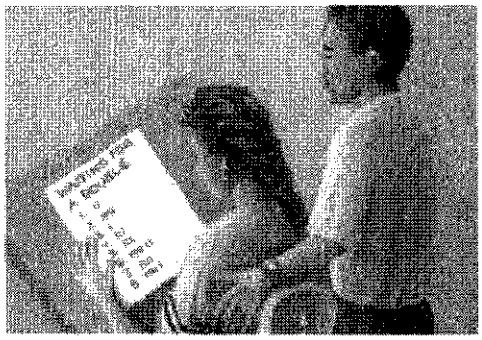
By now you've realized that insurance is rather complicated and involves challenging mathematical and social problems! Although you may have a method for determining a price that you think is fair, you may still be a long way from completely understanding the complex problem of determining the best price. You've likely already decided that to determine the fairest price, you'll have to answer:

What are the chances that my citizen, and other citizens, will need medical care sometime this year?

To answer this question, you're going to have to learn more about the theory of probability. You will start by flipping coins, which is one of the most common ways to investigate ideas about chance and probability. Throughout this section, you will encounter a variety of situations that will help you develop ways of thinking about "what are the chances?" As you'll see, people study probability through both experimentation and theoretical analysis.

Finally, we will return to the main question for this section. You will determine the chances that each citizen will need medical care this year and discuss the social consequences of using mathematics in this context.

Waiting for a Double



In many games that use dice, such as backgammon, you roll two dice at a time. Special rules often apply when you roll a double. (A *double* means the same number is showing on both dice.)

When you play games like these, you might want to know how long it takes to get a double. Here is an experiment to consider:

You roll a pair of dice and continue rolling until you get a double. You record the number of rolls it took to get a double.

Example:

First roll	*Dice show 3 and 4*
Second roll	*Dice show 2 and 5*
Third roll	*Dice show 5 and 3*
Fourth roll	*Dice show 2 and 2*

In this example, it took four rolls to get a double.

Now answer these questions.

1. Predict the average number of rolls it will take to get a double. Write a sentence or two explaining why you made that prediction.
2. Do the experiment ten times. That is, for each experiment, roll a pair of dice until you get a double, counting how many rolls it takes. Write down the number of rolls needed each time.
3. Use the data you gathered in Question 2 to answer these questions.
   a. What was the greatest number of rolls it took to get a double? What was the least?
   b. What was the average of the ten experiments?
4. How close is the average you found in Question 3b to the prediction you made in Question 1? Would you revise your prediction now, based on your experiment? Why or why not?

The Gambler's Fallacy

Introduction

In the game of roulette, a ball is spun around roulette wheel, and it lands in either a red slot or a black slot. (There is also a very small chance that it will land in a green slot, but in this problem, we will simplify things by ignoring that fact.)

The chance of the ball landing in a red slot is the same as its chance of landing in a black slot.

Some gamblers use the following strategy for playing roulette: If the ball lands in red a certain number of times in a row, they bet that black will be next, because they figure it is black's turn. Similarly, if a string of blacks occurs, they bet on red, because they figure red will be more likely than black after a string of blacks.

A Historical Example

In a famous incident in 1913, at the Casino in Monte Carlo, black came up a record *26 times in a row!* By about the fifteenth time, people started betting overwhelmingly on red, believing that it was now red's turn. As a result, the Casino made an enormous sum of money.

The Experiment

Do this experiment with a partner:

Flip a coin 25 times. Record each flip as heads (H) or tails (T), according to the outcome.

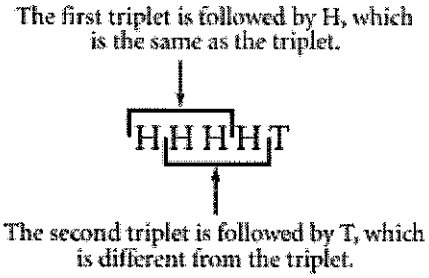
When you have completed all 25 flips, you will have a list of 25 letters, made up of H's and T's.

Starting from the beginning of your list, find the first instance of three identical flips in a row (either three heads or three tails). We will call three identical flips in a row a *triplet*.

Record whether the flip that followed this first triplet was the *same* as the letters in the triplet or *different* from them. Then move to the next triplet. Again, record whether the flip that followed it was the same as or different from the letters in the triplet. Continue in this way through your whole list. (*Note*: If there's a triplet at the end of your 25 flips, ignore it, because nothing follows it.)

Finally, count how many "sames" and how many "differents" you got.

Be Careful! If you have four identical flips in a row, then that is two triplets. For example, suppose you have H H H H T as part of your record. The first three H's form a triplet that is followed by an H, which is the *same* as the triplet. The second, third, and fourth H's also form a triplet, followed by T, which is *different* from the triplet.



Similarly, if you have five identical flips and then a different flip (such as T T T T T H), that gives three triplets. The first two triplets are followed by a flip the *same* as the letters in that triplet. A *different* letter follows the third triplet.

Coincidence or Causation?

Most people accept the idea that a coin doesn't "remember" its previous flips, and therefore the coin's "history" doesn't affect the probability of a given result in the future. But there are times when previous events *do* affect future events.

In each of the following situations, you are to decide whether you think the past will or will not have an influence on the future.

In each case, state your decisions and write a paragraph explaining why you believe that it is correct.

1. A baseball player has averaged hitting a home run once every seven games this season. He has just hit a home run in each of the previous three games. Are his chances of hitting a home run in the next game greater than, less than, or no different from usual?
2. Diabetes is a health condition where the body does not properly process food. It is possible to reduce the risk of developing diabetes by maintaining a healthy diet and exercise routine. Last year, the Center for Disease Control sampled people across various income levels. They found that, on average, the chances of developing diabetes was 9.3%.[1]

   The Center for Disease Control is repeating the sampling procedure this year. However, this year their sample includes many more people in poverty than last year. Would you expect the risk of developing diabetes to be reported at a rate greater than, less than, or no different from last year?
3. The Happy Days Ice Cream Cone Company claims that, on average, only about 1 out of every 100 boxes of their famous ice cream cones will contain a broken cone. The company gladly replaces any box containing a broken cone.

   You purchase a box of Happy Days Ice Cream Cones.

   Upon arriving home, you discover that one of the cones is broken. You decide to return the box and exchange it for a new box. Just to be sure, you immediately check the new box for broken cones. Is the chance that the new box contains a broken cone different from 1 out of 100?

[1] Center for Disease Control (2014): http://www.cdc.gov/diabetes/data/statistics/2014statisticsreport.html

What Are the Chances?

Part I: Finding Probabilities

Sometimes the only way to find the probability of something is to use the observed probability – a model based on using your own experience or an educated guess. In other cases, you can use theoretical probability.

Items A through I pose questions about probability. In each case:

(a) Decide on a probability, using a theoretical model if you can.

(b) Describe how you decided on the probability. State whether your answer was based on a theoretical model or on observed results, or whether it was just a pure guess.

A. You pull one gumball out of a bag that contains three red gumballs, two blue gumballs, and four black gumballs. What is the probability that the gumball you picked is blue?

B. What is the probability of snow falling in Florida at least once next July?

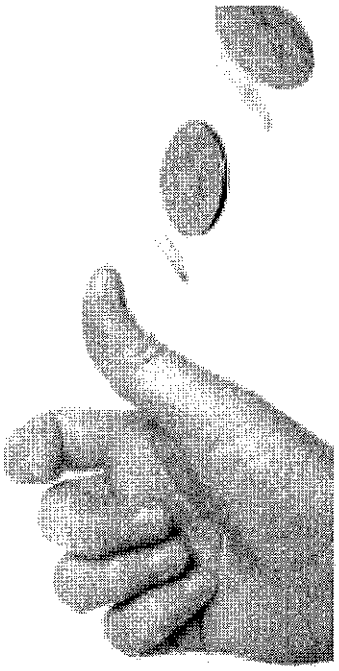
C. You arrive at an intersection with a traffic signal. What is the probability that the light is red?

D. You flip a coin twice. What is the probability of getting one head and one tail?

E. You roll a standard die. What is the probability of getting a prime number?

F. Your teacher selects two students at random from your class to run an errand. What is the probability that you are one of the two students?

G. You randomly point to a student in your mathematics class. What is the probability that this student is wearing sneakers?



H. You roll two dice. What is the probability of getting doubles?

I. You roll a pair of dice until you get doubles. What is the probability that you get doubles in three or fewer rolls?

Part II: Probabilities on the Number Line

Make a number line like the one below. For each item in Part 1, indicate its probability by putting the letter in the proper place on the number line.

Part III: Citizen Probabilities

Consider the question, "what is the probability that your citizen will need medical care this year?"

Decide on a probability that you think is reasonable and place that event on your number line from Part 2.

Ask the Actuary

As you have seen, determining a probability for human behavior can be both difficult and imprecise. In fact, because of this there are people who work as professional **actuaries** to gather and analyze statistics for insurance companies.

Actuaries compile their statistics into something called an actuarial table. The actuarial table has information that helps them to determine the probabilities of life events, ultimately, to use these probabilities to set prices for insurance.

1. Consider your citizen's characteristics (age, gender, income, and health condition) and use the sample actuarial table[1] to determine the probability that your citizen will need medical attention this year. Your method may involve some observed results *and* some estimation.
2. Record the method you used to determine your probability *and* why you think that method makes sense.

Common Medical Procedures - Actuarial Table
All figures are total number of procedures performed for age group during the past year*
Total Population = 300,000

Procedure	*All Ages*	*Under 20*	*20-40***	*40-60*	*Over 60*
Heart Surgery***	75,000	3,000	12,000	20,000	40,000
Cancer Treatment	30,000	2,000	5,000	9,000	11,000
Organ Transplant	15,000	5,000	2,000	3,000	5,000
Diabetes Treatment***	30,000	2,000	6,000	6,000	6,000

*Any person with a family history or pre-existing condition will experience higher risk than stated
**Women aged 20-40 are considered higher risk due to pregnancy
***Rates of medical attention for heart and diabetes are higher for people living in poverty

[1] Mock actuarial table based on data and table format from Center for Disease Control: http://www.cdc.gov/nchs/data/nhds/4procedures/2010pro4_numberprocedureage.pdf

0 to 1, or Never to Always

For each probability given below, think up two situations that have that probability.

In one of those two situations, the probability should be based on a theoretical model. In the other situation, the probability should be based on observed results. Be imaginative!

1. Probability = 0
2. Probability = 2/7
3. Probability = 75%
4. Probability = 1
5. Probability = 2.3
6. Probability = .01

PICTURES OF PROBABILITY

At the beginning of this unit, you were assigned citizen characteristics based on charts like the ones below:

AGE *roll 2 dice and add numbers*	*If the result is* 7, 9	Choose age under 20
	If the result is 2, 5, 6	Choose age 20 – 40
	If the result is 3, 4, 8	Choose age 40 – 60
	If the result is 10, 11, 12	Choose age over 60

ETHNICITY *roll 2 dice and multiply numbers*	*If the result is* 6, 8, 9, 10, 12, 15, 16, 18, 20, 24	White
	If the result is 1, 4, 25	Black
	If the result is 2, 3, 5	Hispanic
	If the result is 30	Asian

Can you be sure that these charts will produce a classroom "country" that is actually representative of the country you live in? Questions like these about probability can be quite challenging! In this section, you will dig deeper into probability to learn more about the question:

How can pictures (or diagrams) help us make sense of probability?

Pictures are important tools for understanding and communicating about mathematics. You have seen that probabilities can be represented with numbers as fractions, percentages, or decimals. You will now use pictures to represent probabilities. Connecting pictures or diagrams to the probabilities they represent will help you work through the activities in this part of the unit. You will blend geometry with probability, creating area models to visualize the probabilities of the possible outcomes for an event.

You will use an area model to find the answers to questions about the results of rolling two dice and, finally, you will use these ideas to answer some important questions about health care and health insurance.

Name:_____ Date:_____

Tree Diagrams

A tree diagram is a way to represent all possible combinations of a situation.

1. Below is a Create-A-Sandwich menu. Create a Tree Diagram that shows all possible combinations (on a ***separate sheet of paper***- MAKE IT NEAT!)

BREAD	MEAT	CHEESE
White	Ham	American
Wheat	Turkey	Swiss
	Beef	Provolone
		Muenster

2. How many total combinations of sandwiches are there? How did you figure that out?

3. What is the probability that you will select a sandwich with white bread?

4. What is the probability that you will select a sandwich with American cheese?

5. What is the probability that you will select a sandwich on wheat bread with ham and any cheese?

6. What is the probability that you will select a sandwich on white bread that has either beef or turkey and has Provolone cheese?

7. What is the probability that the sandwich will be any bread, Turkey and American or Swiss cheese?

8. What is the probability that you will select a sandwich with neither beef nor Muenster cheese?

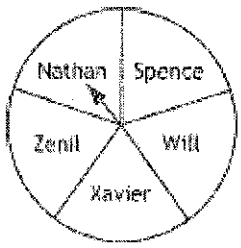
9. What is the probability that you will select a sandwich with Ham given you selected white bread?

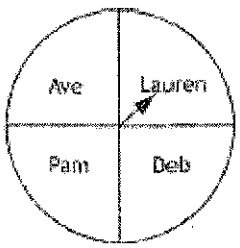
10. What is the probability that you select a sandwich with white bread given it has swiss cheese?

11. What is the probability that you select a sandwich and it does not have beef?

12. What is the probability that you select a sandwich that does not have provolone given it is on wheat bread?

Mr. Sharp is going to pick 1 boy and 1 girl to represent Sprayberry to the local newspaper. Five boys volunteered and 4 girls volunteered. Mr. Sharp wanted to avoid bias so he put the names down on equal spinners to determine who would go. Below are the spinners.





13. Create a tree diagram that shows all the possible combinations of who will be picked. (***On a separate sheet of paper***- MAKE IT NEAT!)

14. How many combinations are 1 boy and 1 girl are there?

15. What is the probability that Nathan will be picked? (leave answer as a percent)

16. What is the probability that Pam will be picked? (leave answer as a percent)

17. What is the probability that Xavier and Lauren will be chosen together? (leave answer as a percent)

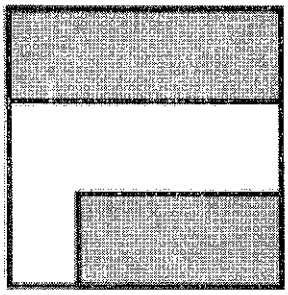
18. What is probability that Will or Zenil will get picked? (leave answer as a percent)

19. What is probability that neither Will nor Zenil gets picked? (leave answer as a percent)

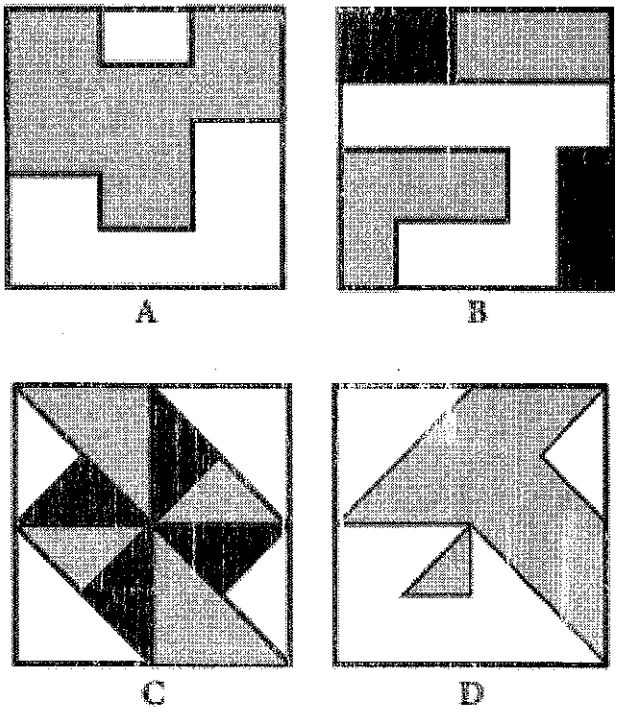
Rug Games

Each diagram in this activity represents a rug. Imagine that a trap door opens directly over the rug. A dart falls down through the door, landing at random somewhere on the rug. In other words, every point on the rug has the same chance of getting hit as every other point.

1. If you were trying to predict which part of this rug will get hit, which color would you choose, gray or white? What is the probability of being hit for each color?



2. For each of rugs A through D, which color do you predict is most likely to be hit? For each color on each rug, find the probability of its being hit.



POW: Linear Nim

Many strategy games involve two players who take turns removing objects from one of several piles according to certain rules. The winner is the person who removes the last object. These games often go by the name Nim.

In one version of Nim, there is only one pile. In this case, the objects can be represented by a single row of marks on a sheet of paper. We will call this game Linear Nim.

Here's how a particular form of Linear Nim works.

At the beginning, there are 10 marks on a sheet of paper.

Each player, in turn, crosses out 1, 2, or 3 of the marks. It doesn't matter which marks are crossed out.

Play continues until all of the marks have been crossed out. The player crossing out the last mark is the winner.

Part I: Finding a Strategy

Your first task is to find a winning strategy for this particular game. You might begin by finding a partner and playing the game for a while.

As you and your partner play, pay attention not only to who wins, but also to when you realize who is going to win and how you know.

The question of who wins may depend on which player goes first. Therefore, one element of your strategy might be to decide whether you want to go first or second.

Be sure the strategy you develop is complete. That is, you should take into account every possible move that your opponent might make.

Part II: Variations

Once you have developed your strategy, investigate how it would have to change if the game were to vary.

This game starts with 10 marks and allows a player to cross out up to 3 marks at a time. What if these numbers were changed? For example, suppose you start with 15 marks or allow a player to cross out up to 4 marks at a time? You can change the initial number of marks, and you can change the maximum per turn that a player can cross out.

How would your strategy change if you varied the game? Are there some cases when you should choose to go first and others when you should choose to go second? What does it depend on?

Consider a variety of examples and look for generalizations.

Write-up

1. *Process*: Describe how you went about understanding the original game and developing a strategy. Indicate the key insights that were important in your understanding.
2. Strategies
   a. Describe the strategy you developed for the original game.
   b. Describe some specific variations you looked at and the strategy you developed for each.
3. *Generalizations*: State any general principles you developed about variations on Linear Nim. In particular, can you describe, in terms of the initial number and the maximum per turn, how to decide whether you'd want to go first or second?
4. *Self-assessment*: Explain what you learned. Be as specific as you can. Assign yourself a grade for your work on this POW. Explain why you think you deserve that grade.

The Counters Game

Each player in this game needs a board with 11 boxes numbered from 2 to 12.

2	3	4	5	6	7	8	9	10	11	12

To start the game, each player puts 11 counters on a board. Players may place counters in the boxes in any way they choose, including putting more than one counter in a single box.

For each turn, a player rolls a pair of dice and then adds the numbers on the dice. Each player who has any counters in the square corresponding to that sum removes *one counter* from that square. Even if a player has more than one counter in that square, only one counter is removed. If a player has no counters in that square, the player does nothing. It doesn't matter who rolls the dice each time.

The winner of the game is the first player to remove all the counters from his or her board.

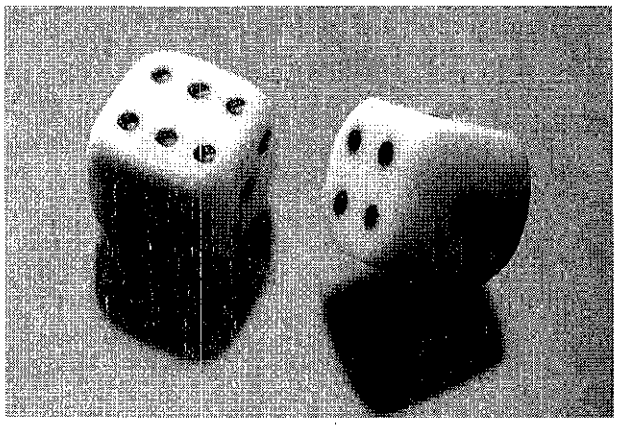
The challenge of the game is choosing where to place the counters so that you will likely be able to remove them quickly during the game.

1. Play one or two practice games with your group. Just guess where to place the counters.
2. Now think about where to place the counters. Write a sentence or two explaining what you think would be a good way to place them and why.
3. Play the game several times with your group, with each player using his or her own strategy for placing the counters.
4. To prepare for a competition among all the groups, discuss with your group the various strategies you used. Choose one strategy for the competition and record what it is.

Rollin', Rollin', Rollin'

Roll a pair of dice 50 times. With each roll, find the sum of the dice. Keep a record of your sums in an organized way.

1. Draw a graph of the data you gathered
2. Write a paragraph about your results. You should summarize your observations about the data and discuss why the results came out the way they did.
3. What new thoughts does this experiment give you about how play the counters game?



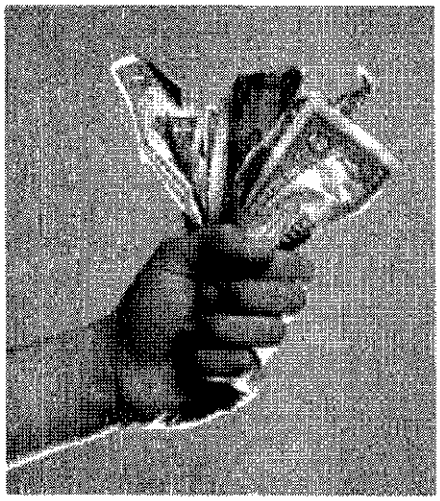
Money, Money, Money

1. Nina was working on the question from *What Are the Chances?* that asks for the probability of getting one head and one tail when a coin is flipped twice.

   She said that because there are exactly three possible outcomes— two heads, one head and one tail, and two tails—the probability of getting one head and one tail is $\frac{1}{3}$.

   Explain why she's wrong. Make your explanation as clear as you can, using diagrams as needed.

2. Imagine that you have two pockets in your jacket. Each pocket contains a $1 bill, a $5 bill, and a $10 bill.

   You reach in and remove one bill from each pocket. Assume that, for each pocket, the three bills are equally likely to be removed.

   a. What are the possible totals of the two bills you take out?

   b. What is the probability that the two bills will total exactly $2?

   c. What is the probability for each of the other possible totals?

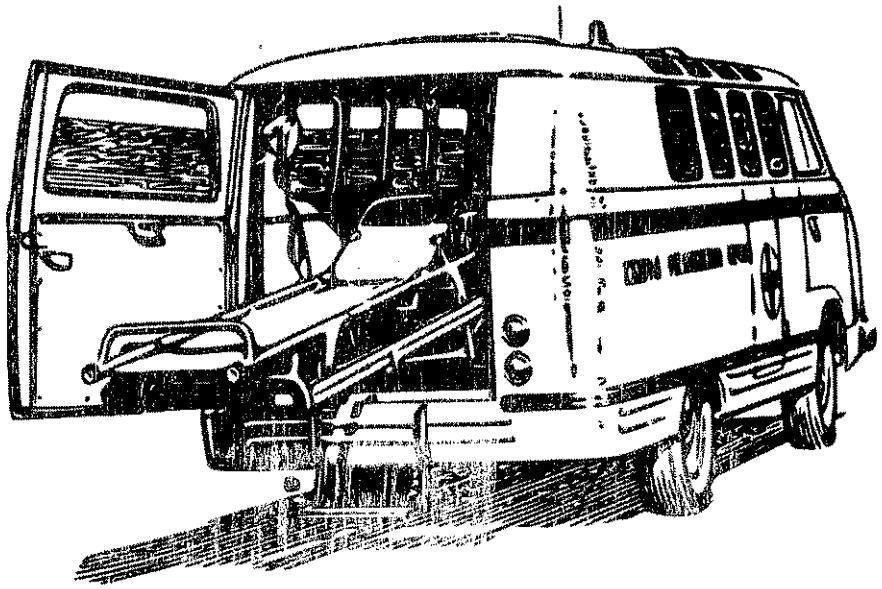
3. What do Questions 1 and 2 have in common? How are these questions related to the problem of two-dice sums?

This AND That

So far, health insurance pricing for citizens has been based on a single health condition. In real life, of course, it is possible that a citizen could be at risk for multiple conditions! What happens when trying to determine the probability that two events will *both* occur? Consider the following scenario.

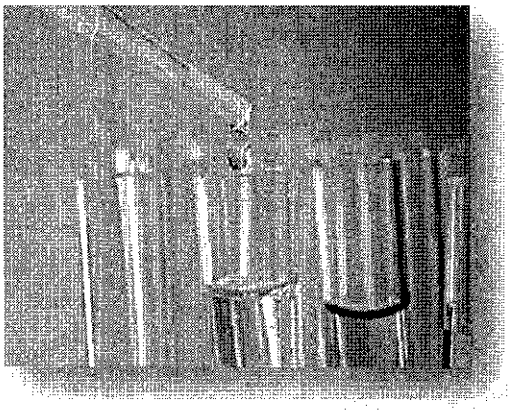
> *Ali is a 40-year-old-man with a heart condition. His probability of needing heart surgery this year is 3/10. However, this is not the only medical condition that could send Ali to the hospital. Statistics show that there is also a 1/10 chance that Ali will have an unexpected medical emergency this year.*

What are the chances that Ali will need heart surgery AND have an unexpected medical emergency?



A False Positive

Part I: Medical Testing



As you have seen, rug diagrams (or area models) can be very helpful in visualizing and working with complex problems of chance.

In health care, medical testing is very important because it can help determine if someone has a specific condition or disease. Naturally, if you are the one being tested, you would want to be sure that the test results were being read correctly.

However, interpreting these test results is not always straightforward and they often involve understanding quite a bit about probability. Because the tests are not 100% accurate, they only return a "correct" reading some of the time. This means that it is possible that your test could report a **false positive** – the test says you have the condition when, in reality, you don't.

Consider the following medical information:

> *The probability of a person having cancer is 1%. Screenings for cancer are said to be 90% accurate. That means that 90% of people who have cancer will test positive and 90% of the people who don't have cancer will test negative.*

Draw a rug diagram to represent this situation. If a person tests positive, how likely is it that they actually have cancer?

(Hint: To answer this question, you might consider a large population of such as 100,000 people. Figure out how many people in that population have cancer and how many of those with and without cancer test positive.)

Part II: A More Realistic Test

The accuracy rate of medical tests is not actually the same for people with and without the condition. Consider the following (more realistic) modification to A False Positive:

> *The probability of a person having cancer is 1%. Screenings for cancer are 80% accurate for those that DO have cancer and are 90% accurate for those that DON'T.*

If a person tests positive, what is the probability that they actually have cancer?

INVESTIGATING PRICE

In recent activities, you have looked at ways of finding the probability of a given outcome, such as flipping two heads in a row or getting a sum of 9 on a pair of dice. You've flipped coins or rolled dice to determine the probability of a particular outcome. You've used rug diagrams to analyze probabilities and you have confirmed some of your results with experimental evidence. All of this has allowed you to determine the likelihood that your citizen will need medical attention this year!

In determining a fair price for health insurance, the situation is complicated by the fact that insurance companies must decide what price to charge you based on the likelihood that you will need medical care, the cost of the procedure, and other factors.

How might someone consider all of those factors to determine a fair price for health insurance? In the next group of activities, you will first look at simple situations to see what happens in "the long run." Eventually, you will draw on those experiences to explore the question:

What is a fair annual price for health insurance for my citizen?

Gradually, you will explore more complex problems. You're moving toward your goal of finding a fair price for health insurance!

Name:_____ Date:_____

EXPECTED VALUE: Winning the Lottery

Millions of people play the lottery in Georgia each year. Using actual odds we will discover what you should expect to make playing the lottery

Georgia Powerball

In this game you pick 5 numbers from 1-69 and one "powerball" number from 1-26.
Below are the possible winning combinations

Combination	**Prize**	**Odds**
Match 5 + Powerball	$40,000,000 (jackpot)	1 in 292,201,338
Match 5	$1,000,000	1 in 11,688,054
Match 4 + Powerball	$50,000	1 in 913,129
Match 4	$100	1 in 36,525
Match 3 + Powerball	$100	1 in 14,494
Match 3	$7	1 in 580
Match 2 + Powerball	$7	1 in 701
Match 1 + Powerball	$4	1 in 92
Just powerball	$4	1 in 28

Assume you are going to play the lottery for an entire year. The drawings for mega millions are twice a week which means you can play 104 times a year. Each play costs $2. What do you predict to be your expected value or in other words how much you expect to win over the year?

To find the Expected value of an event:
STEP ONE: (CASH PAYOUT of event 1)x(PROBABILITY of event 1) + (CASH PAYOUT of event 2)x(PROBABILITY of event 2) + … continue until all events are accounted for. You add all the events together because those are ALL possible outcomes. This will give you the expected value of one play (or of one ticket)

STEP TWO: You had to 'pay to play', that means you need to subtract the COST of the event. In powerball every play costs $2

STEP THREE: **If you want to find the value of MULTIPLE PLAYS**, multiply your answer by the number of times you plan to play (so if you plan on buying 5 lottery tickets multiply by 5)

1. Find the expected value of one ticket in powerball

2. If you played for an entire year what would be your expected value of powerball winnings?

There is an option on Powerball that you can play for higher money by choosing "the POWER PLAY." In this option, you pay an additional $1 per play BUT, it increases your prize money if you win.

Combination	**REGULAR payouts**	**POWER PLAY payouts**	**Odds (same)**
Match 5 + Powerball	$40,000,000 (jackpot)	$40,000,000 (jackpot)	1 in 292,201,338
Match 5	$1,000,000	$2,000,000	1 in 11,688,054
Match 4 + Powerball	$50,000	$100,000	1 in 913,129
Match 4	$100	$200	1 in 36,525
Match 3 + Powerball	$100	$200	1 in 14,494
Match 3	$7	$14	1 in 580
Match 2 + Powerball	$7	$14	1 in 701
Match 1 + Powerball	$4	$8	1 in 92
Just powerball	$4	$8	1 in 28

3. What do you notice about all the prize payouts?

4. Do you think the prizes are high enough to justify paying the extra $1 for the ticket?

5. Find the expected value of one play of powerball if you choose to do the POWER PLAY

6. What is the expected winnings of playing powerball with POWER PLAY for an entire year?

7. Fill in the table

	Powerball	Powerball with POWER PLAY
Expected winnings of one ticket		
Expected winnings for a year		

8. What would you say to someone who wants to play the POWER PLAY because they could double their winnings if they won?

SCRATCH-OFF TICKETS

There are multiple kinds of scratch off tickets that range from $1 to $30. Generally $1 have smaller cash payouts than $30 tickets

9. Do you think you will have a better expected value from a $1 or a $30 ticket? Why?

We are going to investigate a few different scratch -offs for a few different price values

$1 ticket "Shimmering Cash"

To win- Match any of your numbers to the winning number and win the prize shown on that number.

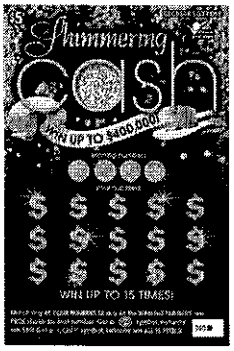
10. Find the expected value of one ticket the $1 Shimmering Cash

Prize	Odds
Free ticket ($1)	1 in 10
$2	1 in 14
$5	1 in 60
$10	1 in 60
$15	1 in 300
$25	1 in 393
$30	1 in 4,000
$50	1 in 2,400
$100	1 in 9,600
$500	1 in 60,000
$6,000	1 in 1,200,000

$5 ticket "Shimmering Cash"

To win- Match any of your numbers to any of the winning numbers and win the prize shown on that number.

11. Find the expected value of playing the $5 shimmering cash



Prize	Odds
Free ticket ($5)	1 in 10
$5	1 in 60
$10	1 in 24
$15	1 in 30
$20	1 in 40
$25	1 in 235
$30	1 in 185
$50	1 in 94
$75	1 in 606
$100	1 in 236
$500	1 in 13,333
$10,000	1 in 1,560,000
$400,000	1 in 1,560,000

HOMEWORK

$30 ticket "SUPER MAX THE MONEY"

Prize	Odds
Free ticket ($30)	1 in 10
$40	1 in 17
$50	1 in 13
$60	1 in 17
$100	1 in 39
$200	1 in 150
$300	1 in 400
$500	1 in 203
$600	1 in 612
$1,000	1 in 3,529
$5,000	1 in 40,000
$50,000	1 in 1,680,000
$200,000	1 in 3,360,000
$10,000,000	1 in 3,360,000

1. Find the expected value of one ticket of Super Max the Money

2. a. Come up with a price for the ticket so that your expected value would be positive.

b. Why did you pick that number?

c. What would your expected value be with that ticket price?

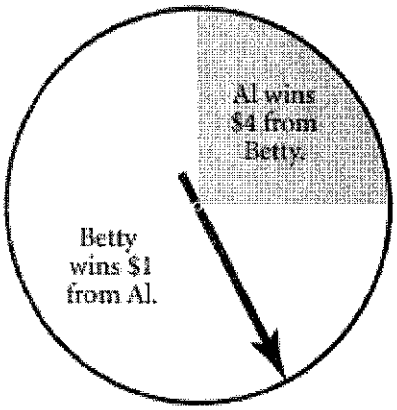
3. Fill in the table below

	$1 Shimmering Cash	$5 Shimmering Cash	$30 Super Max the Money
Expected value of winning with one ticket			
Expected winnings of one ticket if someone gave you the ticket for FREE			

Spinner Give and Take

Al and Betty are playing a game with this spinner.

Each time the spinner comes up in the white area, Betty wins $1 from Al. Each time the spinner comes up in the shaded area, Al wins $4 from Betty.



1. Who will come out ahead over the long run? Write down your prediction and explain your reasoning.

2. Now play the game for 25 spins. Write down your results.

3. If Al and Betty play 100 games, how far ahead is the expected winner likely to be?

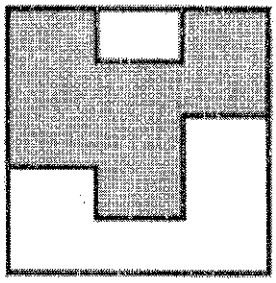
You might make a spinner using a pencil and a paper clip. Bend open one end of the paper clip. Use the pencil to hold the other end in place as you spin the paper clip.

Pointed Rugs

In *Rug Games*, you decided which color in each of the rugs was most likely to be hit by a falling dart. Now you will work with rug diagrams again.

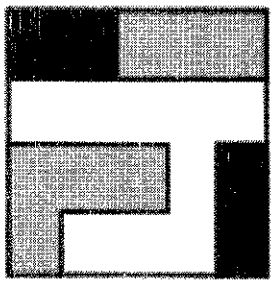
This time, you earn a certain number of points if the dart lands on the color you have chosen. This means that your choice of color involves more than just finding probabilities. You must also take into account the number of points that are awarded each time the dart lands on a certain color.

For each rug, decide which color is the best to bet on to maximize your points in the long run. Write clear explanations to support your answers.



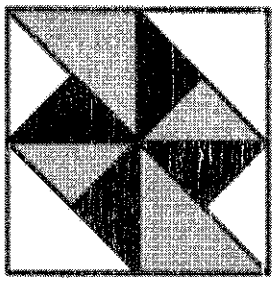
A

Gray: 6 points
White: 8 points



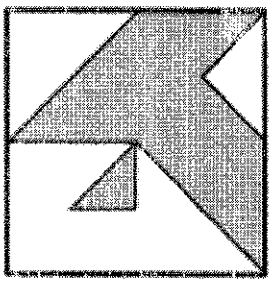
B

Gray: 10 points
White: 8 points
Black: 16 points



C

Gray: 5 points
White: 6 points
Black: 10 points



D

Gray: 15 points
White: 13 points

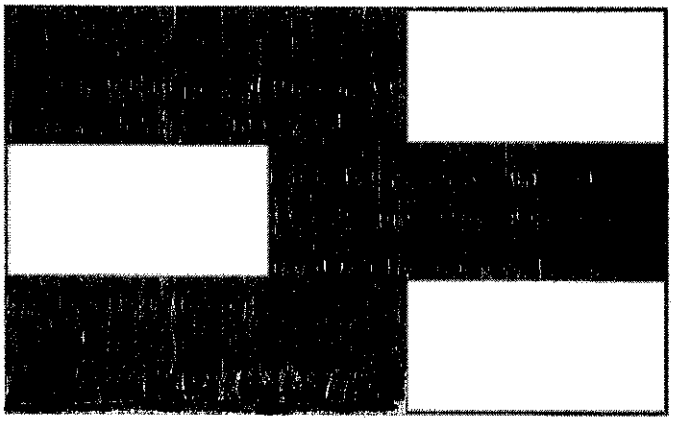
A Fair Rug Game?

1. Tony and Crystal are sitting around this rug watching darts randomly fall from the ceiling.

   If a dart lands on the white part of the rug, Crystal wins $5 from Tony. If a dart lands on the black part, Tony wins $3 from Crystal.

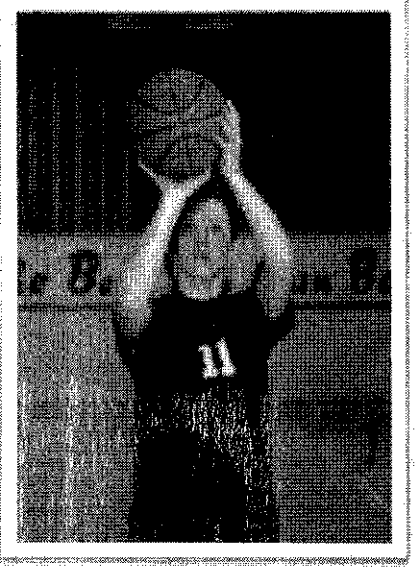
   Do you think this is a fair game? What is Tony's **expected value** for each turn? What is Crystal's? That is, what is each player's average score per turn in the long run?

2. If you think the game is not fair, make it fair by changing the amount of money each player wins. Don't change the rug.



One-and-One[1]

Sometimes in a basketball game, a player is presented with a situation known as a *one-and-one.*



In a one-and-one situation, the player begins by taking a free throw at the basket. If the player misses, that's the end. But if the shot is successful, the player gets to take a second shot.

One point is scored for each successful shot. So the player can end up with 0 points (by missing the first shot), 1 point (by making the first shot but missing the second), or 2 points (by making both shots).

Over many games, Terry has shown that whenever she attempts a free throw, she has about a 60% probability of making it.

In a one-and-one situation, how many points is Terry most likely to score: 0, 1, or 2?

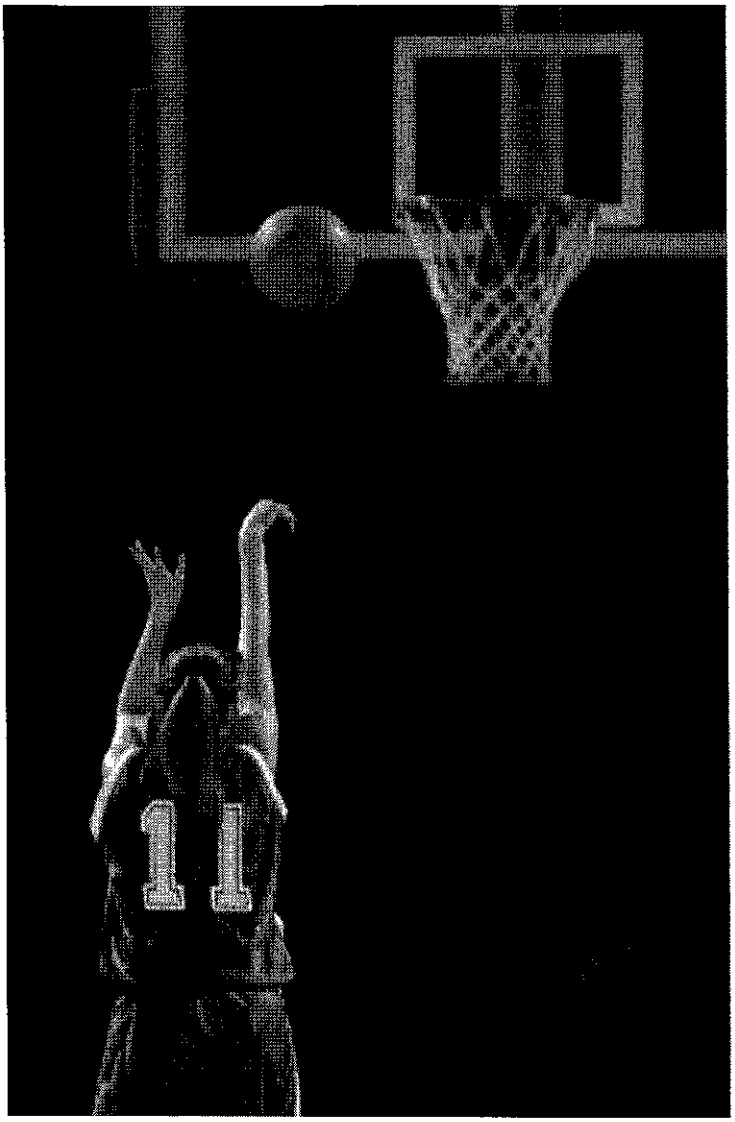
Write down your intuitive guess to this question. Explain why you think that might be the answer.

[1] From Middle Grades Mathematics Project; Probability, Source book by E. Phillips; G. Lappan; J. Winter; W. Fitzgerald

Streak Shooting Shelly

When Streak-Shooting Shelly steps up for a one-and-one situation, her chances of making the first shot are 80%. If she makes her first free throw, there is a 90% chance she will make her second free throw.

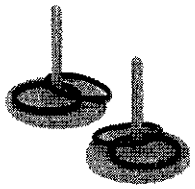
1. In what percentage of one-and-one situations will Shelly score no points? One point? Two points?
2. What is Shelly's expected value per one-and-one situation?



Finding Probability of a Binomial Event (Win/Lose)

WARM UP: Jake has set up a ring toss game. He plans on letting players toss 3

1. How many outcomes are there? List them all out.



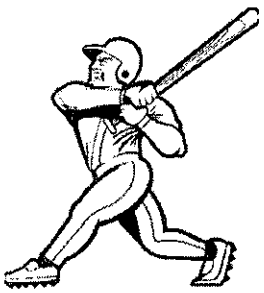
2. What is the probability that someone makes all three tosses?

3. What is the probability that someone makes two tosses?

4. What is the probability that someone makes no tosses? How does that compare to your answer to question 2? Why do you think so?

At the National Baseball Batting Contest, the organizers have set up game booths for the contestants. In the game you will be pitched 5 fastballs and you must hit them into a fair zone to win. The game costs $3 to play. The prizes are as follows

- Hit all 5 pitches, you win a large stuffed animal
- Hit 3 or 4 pitches, you win a small stuffed animal
- Hit 1 or 2 pitches, you win a bat-shaped pencil
- Hit no pitches and you do not win a prize



5. What is the probability of getting each of the following. Leave answers as *unsimplified* fraction (you will see why later)

5 pitches	4 pitches	3 pitches	2 pitches	1 pitch	No pitches

6. The people who run the game are expecting 160 people to play. If 160 people play how many people can they expect to make the following?

5 pitches	4 pitches	3 pitches	2 pitches	1 pitch	No pitches

7. Again assuming that 160 people play the game how many of the following prizes can the workers expect to have to give away?

Large Stuffed Animal	Small Stuffed Animal	Bat Pencil

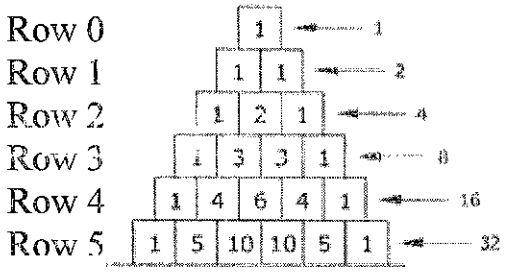
8. The people who run the game know that a large stuffed animal will cost them $6, a small stuffed animal will cost them $1 and a bat pencil costs them $0.25. Assuming 160 people play the game how much money should they expect to spend in prize expenses? Show work or explain how you arrived at this answer.

9. How much profit should the workers expect if 160 people play the game? (remember that is costs $3 to play the game). Show work or explain how you arrived at this answer.

Finding Probability of a Binomial Event (Win/Lose)

PASCAL's TRIANGLE:

What if the game was not 5 pitches but 10 pitches? Would you want to draw a tree diagram to find all the total outcomes? Probably not. Pascal's triangle is a way to find all the probabilities of a binomial event. A **binomial event** is something where there are only 2 outcomes which in this case is hit, miss. Other binomial events could be right/wrong or win/lose.



10. Look at Pascal's triangle above. Each row in the triangle is formed from the one above. Discover the pattern used to create each row.

11. Off to the right hand side is a number. How does that number relate to the number in that row?

12. Look back to your answers for question 6. How do your answers compare to row 5 of Pascal's triangle?

13. Use the pattern to create out to row 8.

Finding Probability of a Binomial Event (Win/Lose)

14. Jackson is creating a bean bag game for carnival day. Each player gets 8 beans bags to throw to try and make it in the hole. Use Pascal's triangle to find the probabilities of the following

0 hits	1 hit	2 hits	3 hits	4 hits	5 hits	6 hits	7 hits	8 hits

He plans to give away the following as prizes

- You get a $5 gift card to Chick-fil-a if you get all 8
- You get a full snickers bar if you make 6-7
- You get a can of coke if you make 4-5
- You do not get anything else for 0-3 hits

15. A can of coke costs $0.75, a snicker's bar costs $1 and obviously the chick-fil-a card costs $5. What is the monetary value (or expected value) of what Jackson will give away in prizes for each player?

16. How much profit will Jackson make per play if he charges $1?

17. Jackson is expected 300 people to play his game, how much profit can he expect?

18. How much should he charge per play in order to make at least $500 in profit (again, assuming 300 people play)

19. Jackson is trying to figure out how many prizes to buy, if he assumes that 300 people will play his game how many of each prize should he buy? Show or explain how you determined that answer

Chick-Fil-A	Snickers	Coke

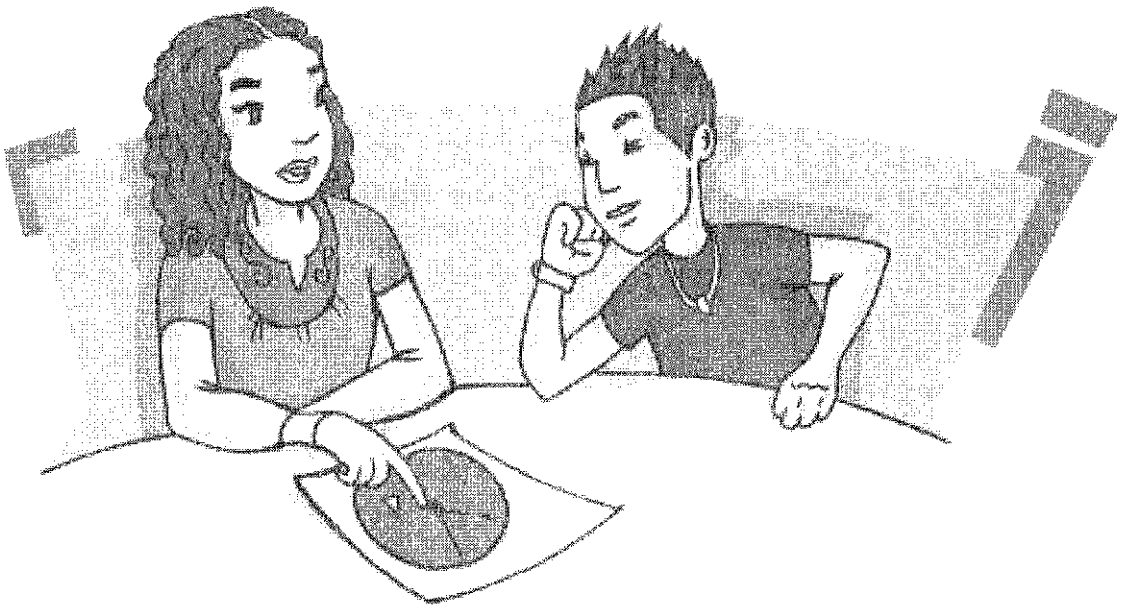
Spins and Draws

1. Al and Betty are playing spinner games again. This time the spinner is divided so that the arrow will land in Al's area $\frac{1}{5}$ of the time and in Betty's area $\frac{4}{5}$ of the time. Al pays Betty 30¢ when the arrow lands in her area. Betty pays Al $1.25 when it lands in his area.
   a. What is Al's expected value per spin? What is Betty's?
   b. How might the payments be changed so that the game is fair?

2. Ari and Brenna are playing a game that involves drawing a card from a standard deck. After each draw, the card is returned to the deck. It doesn't matter which person draws the card—all that matters is which card is drawn.

   If the card drawn from the deck is a jack, Brenna pays Ari 20¢. If the card drawn is a heart, Ari pays Brenna 8¢. If neither a jack nor a heart is drawn, Ari and Brenna each give a penny to charity.

   What is the expected value per draw for Ari? For Brenna? For the charity?



Aunt Zena's Health Insurance

Aunt Zena is in moderately good health. However, according to statistics for her age, she has about a 5% chance of developing a spot of skin cancer each year (you could think of that as a $\frac{1}{20}$ chance).

If she does develop that condition, she would need a $12,000 minor surgery for the removal of the cancerous cells. Aunt Zena knows that she would not be able to afford that surgery if she needed it, and wants to make sure she has insurance to protect her. Her insurance company has decided to charge her $1,000 a year for health insurance.

1. If Aunt Zena continues to pay this price for health insurance, would the insurance company make money, lose money, or break even in the long run?
2. Aunt Zena thinks that insurance company has set an unfair price. She proposes to the insurance company that $500 a year would be a fairer price for her. If the insurance company decides to charge her that price, would the company make money, lose money, or break even in the long run?
3. Maybe Aunt Zena and the health insurance company are *both* wrong? Decide for yourself on what you think a *fair price* would be for Aunt Zena's health insurance. Why do you think that price is fair?

Pricing for Citizens

In previous sections, you determined the probability that your citizen would need yearly medical attention for their particular health condition (heart, diabetes, cancer, or organ transplant).

You might also recall that we know the medical costs for procedures for those various conditions.

Medical Procedure or Treatment	Cost
Heart Surgery	*$150,000*
Diabetes Treatment	*$50,000*
Cancer Treatment	*$100,000*
Organ Transplant	*$300,000*

Consider your citizen's probability of needing care and the cost of their treatment to determine what you think is a fair price for yearly health insurance coverage for them.

A Simpler Problem

In *One-and-One*, Terry took at most two shots at the basket in each one-and-one situation. In *The Carrier's Payment Plan Quandary*, the carrier drew twice from the bag. In each case, you might imagine three turns, a one-and-one-and-one or three draws from the bag.

With health insurance, each of these turns would be a visit to the doctor. The difference is that you never know how many "turns" you'll have. A person could go 5, 10, or even 20 years without needing serious medical attention. This is one reason why determining a fair price for health insurance is so complicated to analyze. And beyond just calculations, we as a community have to consider how health insurance pricing impacts citizens other than ourselves.

In previous sections, you've determined the chances that your citizen will need medical attention during the year. You've also used that probability to calculate their average yearly health expenses (also known as their "expected value"). Now it is time to look beyond your own individual citizen, and decide:

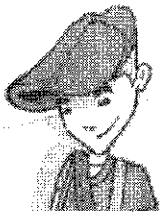
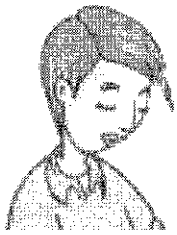
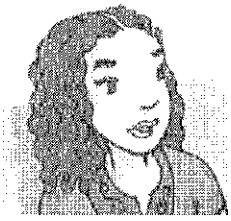
What is a fair way to price health insurance for our community or country?

Often a good approach to solving a complicated problem is to examine a simpler, but related, problem. That's what you will do next, using a simplified version of the *Unit Question*. When you are finished with the simpler problem, you'll apply your new insights to find the fairest price for the original health insurance scenario. Then you'll be ready to wrap up the unit with portfolios and end-of-unit assessments.

A Simpler Country[1]

Imagine a simpler country. In this country, there are only four people and the only medical threat is a deadly disease. Luckily, the disease can be cured with a **$40,000 treatment**. However, even if you get the disease (and are cured), it is still possible for you to contract the disease again!

The four citizens and their chances of sickness are:

Alonso	**Beatriz**	**Claire**	**Daniel**
2% chance of disease each year	16% chance of disease each year	25% chance of disease each year	50% chance of disease each year
			

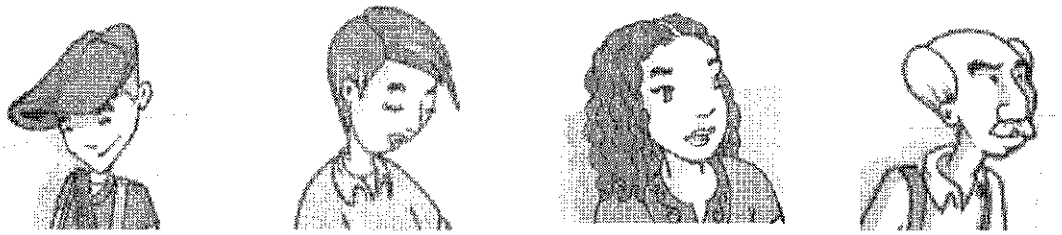
1. If the citizen purchases health insurance, the insurance will pay all of the costs of treatment for the disease. Using what you have learned so far, decide on a yearly health insurance pricing that you think is fair.
2. What factors did you consider while making your pricing decisions? Which factors weighed most heavily in your pricing decisions?
3. Suppose all four citizens live another 50 years. With your health insurance pricing, would the insurance company make money, lose money, or break even?

[1] Based on "Licensed to Ill" by Mathalicious

Everyone Pays the Same

One proposal for health insurance is that everyone pays the same amount for insurance. Let's see what the pros and cons of that approach are by examining our Simpler Country. Remember, the only medical procedure is a **$40,000** disease treatment.

Alonso	**Beatriz**	**Claire**	**Daniel**
2% chance of disease each year	16% chance of disease each year	25% chance of disease each year	50% chance of disease each year



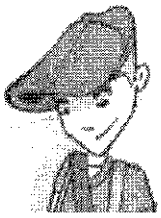
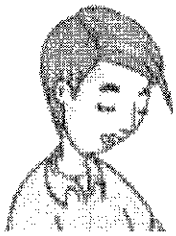
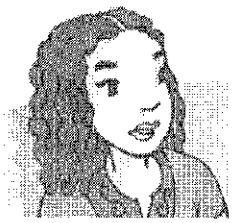
1. On average, how much would each citizen expect to pay for yearly medical expenses in the long run? That is, what is each *citizen's yearly expected value*?
2. On average, how much would the insurance company expect to pay each year to cover medical expenses in the long run? That is, what is the *insurance company's yearly expected value*? Remember, they have to cover expenses for all four citizens.
3. If the insurance company split their yearly expected value evenly across all citizens (charging each citizen the same amount), how much would health insurance cost?
4. At this price, which of the citizens do you think will choose to buy health insurance?
5. If Alonso elects not to buy insurance when he realizes the rates, how would the health insurance company adapt its rates the next year (Year 2)?

Insurance Denied

Part I: Pre-Existing Conditions

As you saw, there were some complications with *Everyone Pays the Same*. To keep costs low for the majority of citizens, some people think insurance companies should deny coverage to citizens who are extremely high-risk. Often, these people have what is called a **pre-existing condition** – meaning that they already have a medical condition (and have been diagnosed as such) prior to signing up for health insurance.

Suppose in the Simpler Country, the insurance company knows that Daniel has a pre-existing condition and refuses to offer him health insurance. Will that make the pricing fair?

Alonso	**Beatriz**	**Claire**	~~**Daniel**~~
2% chance of disease each year	16% chance of disease each year	25% chance of disease each year	~~50% chance of disease each year~~
			

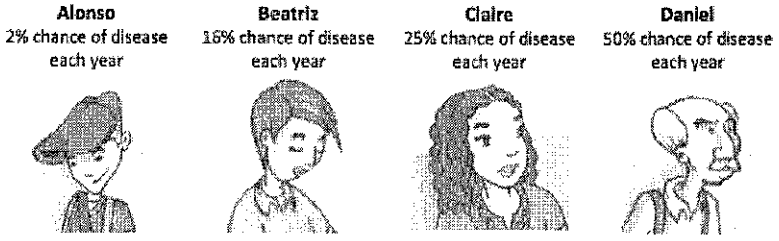
1. What is the health insurance company's expected value? That is, what do they predict their average yearly costs will be?
2. Assume the health insurance company sets rates at their expected value. At this price, which of the citizens do you think will choose to buy health insurance?
3. Do you think this is a better or worse plan than before? Why?

Part II: But Wait...

Even though Daniel is uninsured, he may still get sick with the disease. If he does, the hospital will still treat him with the $40,000 procedure. Because the hospital won't receive payment from the insurance company, they will raise their price for the procedure to make up for their loss in money.

Suppose the next year they raise the price to $54,000. How much would insurance cost for Alonso, Beatriz, and Claire that year?

Safety in Numbers



Part I: Mandatory Coverage

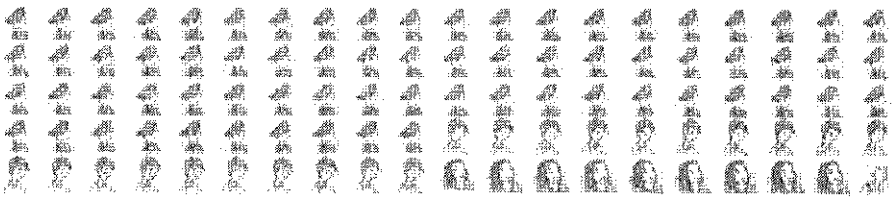
As you have seen, fair pricing for health insurance is complicated – mathematically and ethically. If *Everyone Pays the Same*, some people end up paying more than they would have expected to for health insurance. But with *Pre-Existing Conditions*, some people don't have the right to ensuring their health. It also causes a rise in insurance prices to others.

Some people believe that insurance companies should be required to cover everyone **and** that everyone should be required to have health insurance. This is called *Mandatory Coverage*. Under *Mandatory Coverage*, the first year's prices would be the same as you calculated in *Everyone Pays the Same*. However, because everyone is mandated to buy health insurance every year, the prices will stay the same each year (unlike the situation in *Everyone Pays the Same*).

1. What do you think are the pros and cons of each option – *Everyone Pays the Same*, excluding people with *Pre-Existing Conditions*, and *Mandatory Coverage*?

Part II: A Larger Population

In reality, countries have more than four people. Also, there are more low risk people than high-risk ones. Imagine 70% of the population is like Alonso, 20% is like Beatriz, 9% is like Claire, and 1% is like Daniel.



1. If everyone still has to buy insurance, how much will they pay?
2. What new thoughts do you have about the pros and cons of each option under this larger population?

The Unit Question Revisited

Our original unit question was:

> *Buying health insurance is a way to protect yourself from having to pay for costly medical procedures that become necessary from routine or unexpected health incidents.*
>
> *You pay a certain amount every month, called the* premium, *to the insurance company. Most of the time, if you don't need medical care, the insurance company just takes your money and they pay you nothing.*
>
> *Sometimes, however, you have a medical need. When that happens, the insurance company has to pay your expenses (surgery, visit to the doctor, or whatever the incident is), and they generally have to pay you much more in that month than you paid as a premium. So, you pay a smaller amount each month so that when you have a large medical bill (which you may not be able to afford), the insurance company will pay for you. The cost for medical procedures and treatments to cure the conditions that threaten our "citizens'" health are below. As you can see, they are quite costly!*

Medical Procedure or Treatment	Cost
Heart Surgery	$150,000
Diabetes Treatment	$50,000
Cancer Treatment	$100,000
Organ Transplant	$300,000

> *In the long run, insurance companies take in more money in premiums than they pay out in claims, or they wouldn't be in business.*
>
> *What is a fair price to charge for health insurance?*

1. What information, if any, do you still need in order to answer our unit question?
2. Based on your experiences in this unit, and using all of the information provided, summarize what you think is the fairest price to charge for health insurance in our classroom "country."
3. What lingering questions do you have about health insurance?

Health Insurance and Me

You've worked through the complex problems of mathematically determining a fair price for insurance, as well as the human dilemmas involved in considering what is "fair."

Based on your experiences using probability and expected value to analyze various insurance strategies, as well as the readings and discussions you've had throughout this unit, summarize what you have learned about ensuring a healthy population.

Be sure to include what you think is the best health insurance plan, how you determined the costs, and how you can justify that choice as the most fair option.